Prediction of the Onset of Burnout of Transpiration-Cooled Anodes

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Experiments have shown that a free-burning argon arc with a transpiration-cooled anode has three distinct modes of operation. The present paper describes experiments involving a reversible transition from one of the modes, which does not cause anode ablation, to one of the other modes, which does. The characteristics of each of the modes are discussed. Reasons are postulated for their distinct differences and also for the transition between them. An analysis of the momentum equation for the arc column shows that in the mode of interest, the cathode jet, created by the Lorentz force near the cathode tip, is balanced by the anode jet of transpiring gas. A dimensionless current-blowing parameter is developed for a simplified model that gives an indication of which jet predominates in the arc flow field. Analysis of the experimental data shows that the transition occurs at a critical value of the parameter.

Nomenclature

A = area corresponding to D
$\bar{B}$ = magnetic field strength
D = cathode jet diameter near jet interaction zone
$\bar{E}$ = electric field strength
I = total arc current
$\bar{J}$ = current density
$\dot{m}$ = mass flow rate per unit area of anode transpirant
p = pressure
r = radial coordinate
Re_D = Reynolds number based on D and anode conditions
$\bar{u}$ = velocity vector with components w, u in the r, z directions
V = arc voltage drop
z = axial coordinate
η = dynamic viscosity
μ_0 = permeability of free space
ρ = fluid density
ρ_e = net electric charge density
σ = electrical conductivity

Subscripts

a = anode condition
crit = conditions in arc when transition is impending
D = cathode jet diameter
r = radial
Φ = azimuthal

1. Introduction

TRANSPIRATION-COOLING offers a simple but effective means for increasing the temperature level or energy conversion efficiency of plasma generation systems which use an electric arc discharge as a thermal energy source. In such systems, high net heat fluxes to a surface, such as an electrode, are reduced by blowing a coolant through the surface, which is made of a porous material. The high-energy coolant is combined with the arc-plasma and so the useful energy of the system is increased for a given input.

Previous observations[1,2] have shown that an electric arc with a transpiration-cooled anode has three distinct modes of operation. These correspond in a relative sense to low, medium, and high rates of anode coolant blowing. The first of these, the low-blowing mode, is found to be destructive in nature because operation in this mode leads to rapid anode consumption by ablation and ultimate burnout. On the other hand, the medium-blowing mode is found to provide a safe steady arc operation. Measurements of the rate of ablation[1] show very little anode consumption beyond that which occurs in starting. The high-blowing mode offers anode protection, but the plasma generated in the arc column is found to be in a highly nonequilibrium state with the heavy particles at a relatively low temperature. Application of transpiration cooling to plasma generation systems requires then that the operation be restricted to the medium-blowing mode.

This paper presents a qualitative description of the two basic modes of arc operation associated with the lower blowing rates. An analysis of the momentum equation shows that it is possible to predict, from prior consideration of current and blowing, in which mode the arc will operate. Data are presented which show that the dimensionless momentum parameter developed in the analysis predicts the onset of transition from the medium- to the low-blowing mode for an arc with a fixed geometry. Furthermore, this parameter is shown to correlate arc voltage drop measurements for a variety of operating conditions in the medium-blowing mode.

2. Discussion of the Problem

The anode of an electric arc is subject to high heat loads. The total heat transfer q is given by

$$q = q_r + q_e + q_w + q_c \tag{1}$$

The component q_r is the radiative energy flux and is typically on the order of 2–8% for a short free-burning argon arc near atmospheric pressure.[3] The q_e is the contribution due to the excess kinetic energy of the electrons which is given up as they move from the hot plasma to the relatively cool anode. The q_w is the condensation energy flux, or work function energy, which the electrons give up as they fall into the potential well of the anode. These last two electronic contributions are both proportional to current. Eberhart and Seban[3] showed that they contributed from 20 to almost 100% of the anode energy input for arcs operating in argon over a range of currents from 1090–250 amp at a pressure slightly above atmospheric, depending on arc length. The remaining energy flux q_c is due to convection and is found to be a strong

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function of current, being as large as 72% of the total anode energy at the highest current level in the Eberhart-Seban study. This convective input is due to the cathode jet created by the interaction of the current density with the self-magnetic field in the cathode region where both are large in magnitude. This effect was first explained by Maecker.[4]

Consideration of the four transfer mechanisms shows that if the net heat loss at the anode of an arc is to be reduced there are few alternatives available that do not significantly affect the arc. For a given electrical power input, the radiation term is essentially fixed since the radiating plasma will necessarily be close to the anode. The kinetic energy term will also be fixed. There is a possibility of reducing the condensation energy by providing a low work function surface; however, this is essentially a materials problem and will not be considered further in this study. The convection term does offer some possibilities.

The convective load is caused by the cathode jet which streams along the arc axis as determined near the cathode. Sheer and co-workers[5] avoided the anode convective load in an arc with a transpiration-cooled anode by using noncoaxial electrodes. In this way the hot cathode jet did not impinge on the anode. A further possibility is to use coaxial electrodes and regulate the blowing at the anode so that the cathode jet does not quite reach the surface. This eliminates the convection and, in addition, the anode cooling becomes regenerative in that the transpirant carries the energy picked up from the anode back to the arc plasma region. This system is shown schematically in Fig. 1. In the figure, the cathode jet caused by the electromagnetic forces near the cathode tip is shown streaming upward and meeting the downward streaming anode jet. The shaded portion represents the thermal plasma. The cylindrical core is the current path.

The arc with a transpiration-cooled anode as shown in Fig. 1 has been shown to be similar in most respects to the more typical arc with a water-cooled anode if the blowing is not excessive.[1] If too much blowing is used, the thermal plasma in the arc column region is blown away and an appreciable portion of the interelectrode region is far removed from thermal equilibrium. In this mode, the high-blowing case, the system is not a good producer of thermal plasma and so is not of engineering interest. If the blowing is too little, then the cathode jet penetrates all the way to the anode and the anode is not adequately protected. This is the low-blowing mode. It is the mode of moderate blowing which gives a thermal plasma and a well protected anode. Examples of the latter two modes are shown in Fig. 2. It is important from an engineering standpoint to be able to predict in which mode the arc will operate for given conditions of arc current and transpirant blowing. This is necessary because the anode ablates rapidly in the low-blowing mode (burnout) and the arc then has a limited lifetime. However, in the medium-blowing mode the anode is well protected and the arc can be run for hours with practically no anode ablation other than that encountered during starting.[1]

It is observed in the laboratory that the transition from the medium- to the low-blowing mode and vice versa occurs

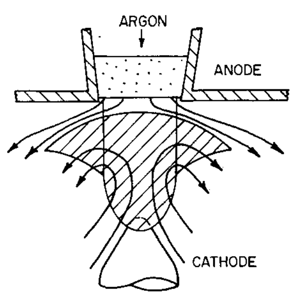

Fig. 1 Schematic view of arc with transpiration-cooled anode.

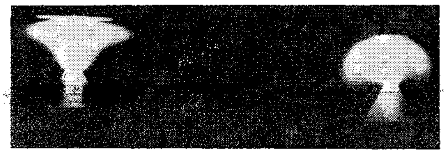

Fig. 2 Arc with transpiration-cooled anode. Left—Low-blowing Mode; right—Medium-blowing Mode.

suddenly (on the order of several hundred milliseconds) and is brought about by the presence of slight instabilities or by slight changes in current or blowing when conditions are near those favorable for transition. Consequently, before an analysis is presented for the prediction of where transition occurs in the range of arc operating variables, some consideration should first be given to the mechanisms at play in such an arc.

Consider an arc that is operating at a relatively high-current density and a moderate blowing rate—an arc, for example, such as the one which appears on the right in Fig. 2. In this mode, spectrographic temperature measurements[1] indicate that the plasma temperature does not vary appreciably with radius in the core region of the arc. In addition, pyrometric measurements indicate that the anode temperature is nearly constant over much of the anode. From these measurements it seems reasonable to assume that the current distribution at the anode is fairly uniform.

Now consider what happens when the anode mass flow is reduced or the current is increased so that the cathode jet can penetrate to the anode surface. There is a sudden increase in the heat load where the cathode jet impinges on the anode and, in addition, a greater portion of the current will flow through this less resistive path. This increased load to the anode center must be balanced by a matching outflow of energy from this region. Part of this accommodation is made by vaporization of the anode material which then shoots off, as does the anode material in the so-called hissing arc or Beck arc. Since this material comes from a spot the size of the cathode jet, it too will form a jet this size. The result then is that most of the electrons enter the anode through the now narrow column and a resulting stream of anode material ejected by the anode collides with the cathode jet. The resulting interaction between the cathode jet and the anode ablative jet gives the wings so evident on the left side of Fig. 2.

3. Analysis of the Problem

The low-blowing mode transition is a transition from one flow domain to another. That is, the mode in which the arc operates is a direct manifestation of the relative strengths of the anode and cathode jets. A parameter is sought which describes this interaction. Above a certain critical value of this parameter the cathode jet will prevail, and thus the low-blowing mode; whereas for values below the critical value, a balance will be established resulting in the medium-blowing mode. The parameter should then be a measure of the stability of the medium-blowing mode.

The problem of isolating such a similarity parameter falls into the realm of dimensional analysis. The most logical approach to dimensional analysis in this case is to write the equation of motion of the system. After proper simplication, the equation is made dimensionless and the reference parameters are grouped into the appropriate terms.

In general, the momentum equation with constant viscosity can be written

$$\rho D\bar{u}/Dt = \bar{F} - \nabla p + \eta \nabla^2 \bar{u} \qquad (2)$$

The use of the constant viscosity form, though questionable here, simplifies the dimensional analysis without affecting the final results, as will be seen. For electromagnetic gas

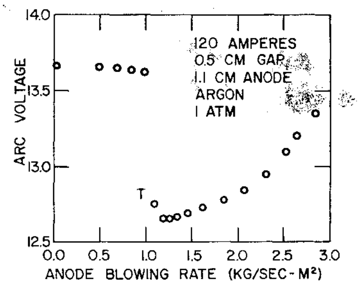

Fig. 3 Voltage-blowing characteristic.

flows, the body force term can be expressed as

$$\bar{F} = \bar{J} \times \bar{B} + \rho_e \bar{E} + \bar{F}_b \quad (3)$$

For free-burning arcs the net charge density ρ_e is zero and for high intensity arcs the buoyancy force $\bar{F}_b$ is negligible. The present concern is with the axial component of the momentum equation because the phenomena of interest, the magnetohydrodynamic cathode jet and the hydrodynamic anode transpirant jet are axially oriented. Then for a circularly symmetric, steady-state arc, Eq. (2) becomes

$$\rho\left(w \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial z}\right) = J_r B_\Phi - \frac{\partial p}{\partial z} + \eta \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2}\right] \quad (4)$$

Now characteristic values of the variables can be introduced to make the equation dimensionless. For the hydrodynamic terms, the reference value of the velocity can be taken as the average anode blowing velocity, reference properties can be taken at the anode surface temperature (experiments show this to be nearly constant over a wide range of conditions), and the characteristic length can be taken as the diameter of the cathode jet. A characteristic pressure can be taken as twice the dynamic pressure of the anode jet. Then the dimensionless hydrodynamic variables can be written

$$u' = u/u_a,\ w' = w/u_a,\ r' = r/D,\ z' = z/D$$
$$p' = p/\rho_a u_a^2,\ \rho' = \rho/\rho_a,\ \eta' = \eta/\eta_a$$

The electromagnetic force term for the axial direction can be made dimensionless in the same way. The radial component of current density can be made dimensionless with the average current density in a uniform conductor of the same cross-sectional area as the cathode jet (more will be said of this later). The azimuthal component of the magnetic field can be made dimensionless with the self-magnetic field at the surface of such a uniform conductor. That is

$$J'_r = J_r/(I/A),\ B'_\Phi = B_\Phi/(\mu_0 I/\pi D)$$

Then Eq. (4) becomes

$$\rho'(w'\partial u'/\partial r' + u'\partial u'/\partial z') = (\mu_0 I^2/\pi \rho_a u_a^2 A)J'_r B'_\Phi - \partial p'/\partial z' + (\eta'/Re_D)[(1/r')(\partial/\partial r')(r'\partial u'/\partial r') + \partial^2 u'/\partial z'^2] \quad (5)$$

The coefficient of the Lorentz force term in Eq. (5) indicates the relative strengths of the cathode and anode jets. That is, if this coefficient is written

$$M = \mu_0 I^2/\pi \rho_a u_a^2 A = \mu_0 \rho_a I^2/\pi \dot{m}^2 A \quad (6)$$

then I^2/A is indicative of the strength of the cathode jet while $\dot{m}^2/\rho_a$ is indicative of the strength of the anode jet. Consequently, a large value of M should presage the low-blowing mode while a small value would indicate the medium-blowing mode. Eq. (6) then forms the basis for the experiments on the transition between modes; if M is indeed a relevant parameter describing the relative importance of the two jets, then some particular value of M should indicate impending transition between modes. It should be noted that an analysis of the problem using the Buckingham Pi Theorem yields the same dimensionless grouping as Eq. (6) within a constant factor of $\pi^2/4$ (Ref. 6). Furthermore, it is interesting to note that the "momentum" number M is similar in many respects to the magnetic force number encountered in the study of magnetohydrodynamic flows where the magnetic field is a parameter. This is defined as[7]

$$S = B_0^2/\mu_0 \rho_0 u_0^2 \quad (7)$$

where the subscript refers to the reference conditions. If in the present problem the magnetic field in Eq. (7) is taken to be the self-magnetic field due to a current I in a uniform conductor of cross section A, then Eqs. (6) and (7) are the same to within a constant factor.

4. Experimental Results

The electric arc experiments were performed using the apparatus described in Ref. 6. The anode was a porous 1.1 cm-diam graphite plug (No. 60 porous graphite, United Carbon Company) mounted in a water-cooled holder and the cathode was a 0.953 cm diameter 2% thoriated tungsten rod ground to a 60° cone at one end and mounted in a water-cooled holder at the other. The arc apparatus was mounted in a water-cooled chamber and the arc environment was argon at 760 mm Hg pressure. The transpirant gas was also argon. The electrode spacing was maintained at 0.5 cm.

An experimental run was initiated by starting the arc at a low current and a high-blowing rate (medium-blowing mode). Then the current was adjusted to the desired value for the run and the blowing was slowly reduced in small steps until a transition occurred to the low-blowing mode. The result for each run was a data set that when plotted looked like Fig. 3. The input information for calculating M was then taken from the data point just before transition as the blowing was reduced (point T in Fig. 3).

There is some question as to what area and diameter of the cathode jet should be used in developing Eq. (5). These quantities, indicating the strength of the cathode jet, are used in making the current density and self-magnetic field dimensionless and they do not represent parameters of the problem. Since a relevant dimensionless parameter should contain only independent variables of the problem, the analysis must be extended to yield A as a function of these other variables. This can be achieved by introducing a simple intuitive model which has been proposed by Finkelnburg and Maecker.[9]

The core region of a free-burning arc is essentially the developed cathode jet. Temperature measurements[1] indicate that in the core of an arc with a transpiration-cooled anode, the isotherms are quite similar to those in arcs with conventional water-cooled anodes. That is, in the region near the anode where the cathode and anode jets impinge on one another, the temperature does not vary appreciably in the radial direction. Since the electrical conductivity at temperatures greater than about 10^4 °K is not a strong function of temperature, it is not unreasonable to assume that the current carrying core of the free-burning arc is a region of nearly uniform conductivity. In addition, it is well known that the electric field strength does not vary with radius. That the current density is uniform near the transpiration-cooled anode has, in fact, been experimentally verified.[10] Thus the definition of current density and Ohm's law yield for the current

$$I = \bar{J} \cdot \bar{A} = (\sigma \bar{E}) \cdot \bar{A}$$

so that as an approximation for the thermal arc regime

$$A = C_1 I \tag{8}$$

The constant C_1 in Eq. (8) can be taken from data available in the literature or it can be deduced from experiments. The latter approach was the one taken in this investigation.

If M is constant at transition, call it M_{crit}, then the product $(M_{crit}A)$ should be a linear function of current. That is

$$M_{crit}A = \mu_0 \rho_a I^2/\pi \dot{m}^2 = C_2 I \tag{9}$$

at least to within an additive constant. The data in Fig. 4 indicate that $(M_{crit}A)$ does indeed vary linearly with I but only out to a point. After that, $(M_{crit}A)$ appears to be a constant, although the data scatter at large currents may make this statement presumptuous.

Consideration of this result shows that the area A in Eq. (9) increases with current up to the point where the area of the current-carrying cathode jet equals the porous anode area. After that, the area remains constant at the anode area. That this is reasonable is shown by experiment. The arc attaches to the anode holder only when it becomes unstable. That is, increasing the current to large values does not make the arc get larger than the porous plug.

If this is so, then a best fit line with a uniform slope through the low current data should intersect a zero slope best fit line at a value of $(M_{crit}A)$ corresponding to the anode area. Figure 4 indicates that this happens at a current of about 174 amp. From this data then, the cathode jet area at the point of jet interaction just below the anode, the most relevant area for use in Eq. (6), is

$$A = (5.24I + 38.34) \times 10^{-7}\text{m}^2, I < 174 \text{ amp} \tag{10}$$

and

$$A = 0.950 \times 10^{-4}\text{m}^2, I > 174 \text{ amp} \tag{11}$$

Experiments show that the anode temperature over the point of arc attachment is uniform and varies only slightly with current and blowing in the medium-blowing mode near transition. This temperature was measured by pyrometer and was found to be 2950°K as averaged over many readings.

The density of the anode transpirant, if assumed to be at this temperature, is 0.165 kg/m³. Table 1 shows the values of M_{crit} over the range of current and blowing for this experiment. The average value of the measured M_{crit} is 16.1 with a standard deviation of 1.0. If the one most deviant point is excluded, the standard deviation improves to 0.6. It appears then that this parameter does correlate the transition

Table 1 Critical value of the momentum parameter

I, crit, amp	$\dot{m}$, kg/sec-m²	M_{crit}
90	0.83	15.1
100	0.87	15.3
110	0.91	15.5
120	0.96	15.6
120	0.94	16.3
130	0.98	16.2
140	1.02	16.1
150	1.10	14.8
150	1.06	16.0
160	1.08	16.5
180	1.21	15.5
180	1.19	16.0
195	1.25	17.0
210	1.41	15.3
210	1.37	16.2
225	1.50	16.5
240	1.54	16.9
270	1.62	19.2

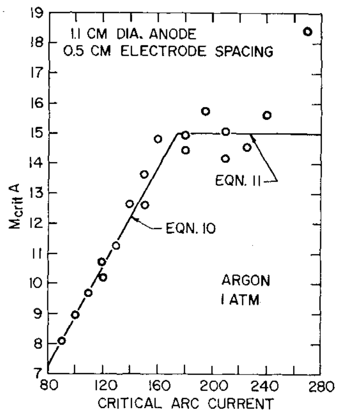

Fig. 4 Dependence of A on current.

data for a fixed geometry arc with a transpiration-cooled anode.

Figure 5 implies the square root dependence indicated by Eq. (9), though inconclusively due to the limited range in the data. In fact, below 174 amp an empirical square root curve fits the data very well, whereas above this current the severe data scatter evidenced by Fig. 4 makes it difficult to draw any conclusions.

The parameter M appears to be significant to the problem of an arc with a transpiration-cooled anode, at least as far as the medium-blowing mode is concerned, and this is the mode of engineering interest. If M is indeed important to the problem, it should also be useful in correlating the dependent variables of the problem such as the voltage drop across the arc.

It was decided to make the voltage dimensionless with the voltage nearest transition in the medium-blowing mode to compensate for differences in experimental conditions. The results of this investigation and Ref. 8 are shown in Fig. 6. Here the ratio of voltage drop to voltage drop at transition is plotted against M^{-1}. This figure includes 61 data points from the present work (6 runs such as shown in Fig. 3) and 70 points from Ref. 8 (6 runs). Only six data points lie to the right of $M^{-1} = 1.0$. One obviously bad point near the left of the graph was excluded in the drawing of the data envelope. The agreement, while not outstanding, indicates that the parameter M is basic to the problem at hand.

5. Conclusions

The momentum parameter as described by Eq. (6) appears to be significant for prediction of the onset of transition from a useful mode to a burnout mode for a fixed geometry arc with a transpiration-cooled anode. This parameter has a critical value of 16.1 at transition for the range of values of current and blowing used in this experiment. For a particular experiment then, if M is greater than 15 or 16, the cathode jet

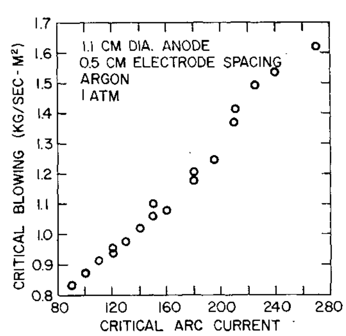

Fig. 5 Relation between blowing and current for transition.

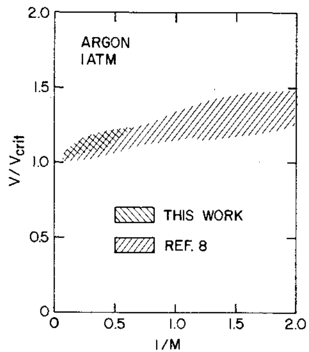

Fig. 6 Voltage correlation for the medium-blowing mode.

should predominate and the arc will operate in the low-blowing mode and the anode will ablate. If M is less than this value, the arc will operate in the medium-blowing mode with very little ablation of the anode.

Attempts to correlate the voltage drop data for the medium-blowing mode were only partially successful. The reason for the lack of excellent agreement may lie in the factor used to make the voltage dimensionless. The somewhat arbitrary choice of the voltage drop near transition may well be a poor choice.

It is possible that the critical value of M could be predicted using the method of small disturbances in conjunction with Eq. (5). However, such an analysis is beyond the scope of this paper.

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